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The second hyper-zagreb indices and coindices of disjunction and symmetric difference of graphs

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Abstract
A topological index is a numerical descriptor of a molecule, based on a certain topological feature of the corresponding molecular graph. In this paper, we explore here some basic mathematical properties and present explicit formulas for the second Hyper-Zagreb coindex under graph operations (disjunction and symmetric difference).

Keywords: Second hyper-zagreb index, second hyper-zagreb coindex, graph operations, disjunction, symmetric difference

Introduction
A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix which represent the whole graph, and these representations are aimed to be uniquely defined for that graph. Topological index and coindex are a numeric quantity with a graph which characterizes the topology of the graph and are invariant under graph automorphism [1, 2]. The methods on topological index and coindex computation are very suitable and serviceable for developing countries in which they can yield available medical information about new drugs without chemical experiment [3, 4].

All graphs in this paper are finite and simple, let *G* be a finite simple graph on $V(G) = n,$ vertices and $E(G) = m,$ edges, the degree of a vertex v is the number of edges incident to v, denoted by $\delta_G(v)$. The first and second Zagreb indices have been introduced by Gutman and Trinajestić in 1972 [5]. They are defined as:
Trinajestić in 1972 [5]. They are defined as:

$$M_1(G) = \sum_{v\in V(G)} \delta_G{}^2(v) = \sum_{uv\in E(G)} [\delta_G(u) + \delta_G(v)],$$

$$M_2(G) = \sum_{uv\in E(G)} \delta_G(u)\delta_G(v)$$

The first and second Zagreb coindices have been introduced by A.R. Ashrafi, *et al.* in 2010 [6]. They are defined as:

$$\overline{M}_1(G) = \sum_{uv\notin E(G)} [\delta_G(u) + \delta_G(v)],$$

$$\overline{M}_2(G) = \sum_{uv\notin E(G)} [\delta_G(u)\delta_G(v)$$

In 2013, G.H. Shirdel, H. Rezapour and A.M. Sayadi [7]. Introduced degree-based of Zagreb indices named Hyper-Zagreb index which is defined as:

$$HM(G) = \sum_{uv\in E(G)} (\delta_G(u) + \delta_G(v))^2,$$

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In 2014, G. B. A. Xavier *et al.* [8]. re-defined the generalized version for Zagreb indices, the third Zagreb indices for a graph G defined as:

$$ReZG_3(G) = \sum_{uv\in E(G)} \delta_G(u)\delta_G(v)[\delta_G(u) + \delta_G(v)],$$

In 2016, Maryam Veylaki *et al*. [9]. introduced distance-based of Zagreb indices named Hyper-Zagreb coindex which is defined as:

$$\overline{HM}(G) = \sum_{uv\notin E(G)} (\delta_G(u) + \delta_G(v))^2,$$

In 2016, Wei Gao *et al*. [10] defined new version of Zagreb topological indices, based on the Hyper-Zagreb index that defined as above. Then, the Second Hyper-Zagreb index of a graph G, which is defined as the sum of the weights $(\delta_G(u)\delta_G(v))^2$ and is equal to:

$$HM_2(G) = \sum_{uv\in E(G)} \left(\delta_G(u)\delta_G(v)\right)^2,$$

Furtula and Gutman in 2015 introduced forgotten index (F-index) [11]. which defined as:

$$F(G) = \sum_{v\in V(G)} \delta_G{}^3(v) = \sum_{uv\in E(G)} [\delta_G{}^2(u) + \delta_G{}^2(v)],$$

N. De, S.M.A. Nayeem and A. Pal. in 2016 defined forgotten coindex (F-coindex) [12]. which defined as:

$$\bar{F}(G) = \sum_{uv\notin E(G)} [\delta_G{}^2(u) + \delta_G{}^2(v)],$$

In 2020, computed exact formulas for the Y-index of some graph operations by A. Alameri *et al*. [13]. They defined a new degree-based of Zagreb indices named Y-index or" Yemen index" defined as:

$$Y(G) = \sum_{v\in V(G)} \delta_G{}^4(v) = \sum_{uv\in E(G)} [\delta_G{}^3(u) + \delta_G{}^3(v)],$$

Also, A. Alameri *et al* in 2020, introduced Y-coindex [14] defined as:

$$\bar{Y}(G) = \sum_{uv\notin E(G)} [\delta_G{}^3(u) + \delta_G{}^3(v)]$$

Here, we define a new version of Zagreb topological indices, based on the Hyper-Zagreb index that defined as above. Then, the Second Hyper-Zagreb index of a graph *G*, which is defined as the sum of the weights $(\delta_G(u)\delta_G(v))^2$ such that $uv \notin E(G)$ and is equal to [15]:

$$\overline{HM}_2(G) = \sum_{uv\notin E(G)} \left(\delta_G(u)\delta_G(v)\right)^2$$

1. Preliminaries

In this section we give basic and preliminary concepts which we shall use later [16]. To unexplained terminology, we refer the interest reader to [4].

Definition 2.1

- The disjunction $G_1 \vee G_2$ of graphs G_1 and G_2 is the graph with vertex set $V(G_1) \times V(G_2)$ and (u_1, v_1) is adjacent with (u_2, v_2) whenever $u_1u_2 \in E(G_1)$ *or* $v_1v_2 \in E(G_2)$.
- The symmetric difference $G_1 \oplus G_2$ of two simple and connected graphs G_1 and G_2 is the graph with vertex set $V(G_1) \times V(G_2)$ and $E(G_1 \oplus G_2) = \{(u_1, u_2)(v_1, v_2) | u_1v_1 \in E(G_1)$ or $u_2v_2 \in E(G_2)$ but not both$\}$.

Lemma 2. 2: Let G_1 and G_2 be graphs with $|V(G_1)| = n_1$, $|V(G_2)| = n_2$, $|E(G_1)| = m_1$, and $|E(G_2)| = m_2$, then

a. $\delta_{G_1\vee G_2}(u, v) = n_2\, \delta_{G_1}(u) + n_1\, \delta_{G_2}(v) - \delta_{G_1}(u)\delta_{G_2}(v)$,

b. $\delta_{G_1\oplus G_2}(u, v) = n_2\, \delta_{G_1}(u) + n_1\, \delta_{G_2}(v) - 2\delta_{G_1}(u)\delta_{G_2}(v)$,

Proposition 2.3: Let G_1 , G_2 be two graphs with n_1, n_2 vertices and m_1, m_2 edges, respectively, then

$$M_1(G_1 \vee G_2) = (n_2{}^3 - 4m_2n_2)M_1(G_1) + M_1(G_2)M_1(G_1) + (n_1{}^3 - 4m_1n_1)M_1(G_2) + 8m_1m_2n_1n_2,$$

a.

b.

$$M_2(G_1 \vee G_2) = ((n_1{}^2 - 2m_1)^2 - 2n_1{}^2m_1)M_2(G_2) + ((n_2{}^2 - 2m_2)^2 - 2n_2{}^2m_2)M_2(G_1) + (2n_1{}^2n_2m_1 - 4m_1{}^2n_2)M_1(G_2) + (2n_2{}^2n_1m_2 - 4m_2{}^2n_1)M_1(G_1) - n_1n_2M_1(G_2)M_1(G_1) + 2n_2M_2(G_1)M_1(G_2) + 2n_1M_2(G_2)M_1(G_1) - 2M_2(G_2)M_2(G_1) + 4m_2m_1(n_2{}^2m_1 + n_1{}^2m_2),$$

c.

$$M_1(G_1 \oplus G_2) = (n_1n_2{}^2 - 8m_2n_2)M_1(G_1) + 4M_1(G_1)M_1(G_2) + (n_2n_1{}^2 - 8m_1n_1)M_1(G_2) + 8m_1m_2n_1n_2,$$

d.

$$M_2(G_1 \oplus G_2) = ((n_1{}^2 - 2m_1)^2 - 4n_1{}^2m_1)M_2(G_2) + ((n_2{}^2 - 2m_2)^2 - 4n_2{}^2m_2)M_2(G_1) + (2n_1{}^2n_2m_1 - 8m_1{}^2n_2)M_1(G_2) + (2n_2{}^2n_1m_2 - 8m_2{}^2n_1)M_1(G_1) - 2n_1n_2M_1(G_2)M_1(G_1) + 8n_2M_2(G_1)M_1(G_2) + 8n_1M_2(G_2)M_1(G_1) - 16M_2(G_2)M_2(G_1) + 4m_2m_1(n_2{}^2m_1 + n_1{}^2m_2).$$

e.

Theorem 2.4: Let G_1 , G_2 be two simple graphs with n_1, n_2 vertices and m_1, m_2 edges, respectively, then

$$HM_2(G_1 \vee G_2) = HM_2(G_2)\Big[n_1\Big(n_1{}^5 + 16n_1m_1{}^2 - 10n_1{}^3m_1 - 8n_1M_2(G_1) - 2n_1F(G_1) + 4ReZG_3(G_1)\Big) + M_1(G_1)(M_1(G_1) + 6n_1{}^3 - 8n_1m_1)\Big] + n_2{}^2F(G_2)M_1(G_1)[n_1{}^3 - 4n_1m_1 + M_1(G_1)] + HM_2(G_1)\Big[n_2\Big(n_2{}^5 + 16n_2m_2{}^2 - 10n_2{}^3m_2 - 8n_2M_2(G_2) - 2n_2F(G_2) + 4ReZG_3(G_2)\Big) + M_1(G_2)(M_1(G_2) + 6n_2{}^3 - 8n_2m_2)\Big] + n_1{}^2F(G_1)M_1(G_2)[n_2{}^3 - 4n_2m_2 + M_1(G_2)] + 2n_2ReZG_3(G_2)\big[n_1{}^2\big(2n_1{}^2m_1 + F(G_1) - 8m_1{}^2 + 4M_2(G_1)\big) - M_1(G_1)(M_1(G_1) + 2n_1{}^3 - 6n_1m_1)\big] + 4n_2{}^2M_2(G_2)[4n_1{}^2m_1{}^2 + M_1{}^2(G_1) - 4n_1m_1M_1(G_1)] + 2n_1ReZG_3(G_1)\big[n_2{}^2\big(2n_2{}^2m_2 + F(G_2) - 8m_2{}^2 + 4M_2(G_2)\big) - M_1(G_2)(M_1(G_2) + 2n_2{}^3 - 6n_2m_2)\big] + 4n_1{}^2M_2(G_1)[4n_2{}^2m_2{}^2 + M_1{}^2(G_2) - 4n_2m_2M_1(G_2)] + 2M_1(G_1)M_1(G_2)\big[n_1{}^3\big(2n_2m_2 - M_1(G_2)\big) + n_2{}^3\big(2n_1m_1 - M_1(G_1)\big)\big] + n_1{}^4m_1M_1{}^2(G_2) + n_2{}^4m_2M_1{}^2(G_1) - 6n_1n_2ReZG_3(G_1)ReZG_3(G_2) - n_1{}^2n_2{}^2[F(G_1)F(G_2) + 8M_2(G_1)M_2(G_2)] - 2HM_2(G_1)HM_2(G_2).$$

$$HM_2(G_1 \oplus G_2) = HM_2(G_2)\Big[n_1\Big(n_1^5 + 64n_1m_1^2 - 20n_1^3m_1 - 64n_1M_2(G_1) - 16n_1F(G_1) + 64ReZG_3(G_1)\Big) + M_1(G_1)(16M_1(G_1) + 24n_1^3 - 64n_1m_1)\Big] + n_2^2F(G_2)M_1(G_1)[n_1^3 - 8n_1m_1 + 4M_1(G_1)] + HM_2(G_1)\Big[n_2\Big(n_2^5 + 64n_2m_2^2 - 20n_2^3m_2 - 64n_2M_2(G_2) - 16n_2F(G_2) + 64ReZG_3(G_2)\Big) + M_1(G_2)(16M_1(G_2) + 24n_2^3 - 64n_2m_2)\Big] + n_1^2F(G_1)M_1(G_2)[n_2^3 - 8n_2m_2 + 4M_1(G_2)] + 4n_2ReZG_3(G_2)\big[n_1^2\big(2n_1^2m_1 + 2F(G_1) - 8m_1^2 + 8M_2(G_1)\big) - M_1(G_1)(4M_1(G_1) + 2n_1^3 - 12n_1m_1)\big] + 16n_2^2M_2(G_2)[n_1^2m_1^2 + M_1^2(G_1) - 2n_1m_1M_1(G_1)] + 4n_1ReZG_3(G_1)\big[n_2^2\big(n_2^2m_2 + 2F(G_2) - 8m_2^2 + 8M_2(G_2)\big) - M_1(G_2)(4M_1(G_2) + 2n_2^3 - 12n_2m_2)\big] + 16n_1^2M_2(G_1)[n_2^2m_2^2 + M_1^2(G_2) - 2n_2m_2M_1(G_2)] + 4M_1(G_1)M_1(G_2)\big[n_1^3\big(n_2m_2 - M_1(G_2)\big) + n_2^3\big(n_1m_1 - M_1(G_1)\big)\big] + n_1^4m_1M_1^2(G_2) + n_2^4m_2M_1^2(G_1) - 48n_1n_2ReZG_3(G_1)ReZG_3(G_2) - 2n_1^2n_2^2[F(G_1)F(G_2) + 8M_2(G_1)M_2(G_2)] - 64HM_2(G_1)HM_2(G_2).$$

Theorem 2.5: Let G_1 , G_2 be two simple graphs with n_1, n_2 vertices and m_1, m_2 edges, respectively, then

a. $$Y(G_1 \vee G_2) = n_1Y(G_2)[n_1^4 + 6n_1M_1(G_1) - 8n_1^2m_1 - 4F(G_1)] + 4n_1^2n_2F(G_2)[2n_1m_1 - 3M_1(G_1)] + n_2Y(G_1)[n_2^4 + 6n_2M_1(G_2) - 8n_2^2m_2 - 4F(G_2)] + 4n_2^2n_1F(G_1)[2n_2m_2 - 3M_1(G_2)] + Y(G_1)Y(G_2) + 12n_1n_2F(G_1)F(G_2) + 6n_1^2n_2^2M_1(G_1)M_1(G_2),$$

b. $$Y(G_1 \oplus G_2) = n_1Y(G_2)[n_1^4 + 24n_1M_1(G_1) - 16n_1^2m_1 - 32F(G_1)] + 8n_1^2n_2F(G_2)[n_1m_1 - 3M_1(G_1)] + n_2Y(G_1)[n_2^4 + 24n_2M_1(G_2) - 16n_2^2m_2 - 32F(G_2)] + 8n_2^2n_1F(G_1)[n_2m_2 - 3M_1(G_2)] + 16Y(G_1)Y(G_2) + 48n_1n_2F(G_1)F(G_2) + 6n_1^2n_2^2M_1(G_1)M_1(G_2),$$

2. Main results

In the following section, we study the second Hyper-Zagreb coindex of disjunction $G_1 \vee G_2$ and symmetric difference $G_1 \oplus G_2$, will be explained, we consider a finite simple connected graph G with vertex and edge sets $V(G)$, and $E(G)$, respectively. For a graph G, the degree of a vertex u is the number of edges incident to u, denoted by $\delta_G(u)$. The complement of G, denoted by $\overline{G}$, is a simple graph on the same set of vertices $V(G)$ in which two vertices u and v are adjacent, i.e., connected by an edge uv, if and only if they are not adjacent in G. Hence, $uv \in E(\bar{G})$, if and only if $uv \notin E(G)$. Obviously $E(G) \cup E(\bar{G}) = E(K_n)$, and $\overline{m} = E(\overline{G}) = \binom{n}{2} - m$, the degree of a vertex u in $\overline{G}$ is the number of edges incident to u, denoted by $\delta_{\overline{G}}(u) = n - 1 - \delta_G(u)$.

Proposition 3.1: Let G be a graph with n vertices and m edges. Then.

$$\overline{HM}_2(G) = \frac{1}{2} M_1^2(G) - \frac{1}{2}Y(G) - HM_2(G)..$$

Theorem 3.2: The Hyper-Zagreb coindex of $G_1 \vee G_2$ is given by:

$$\overline{HM}_2(G_1 \vee G_2) = \frac{1}{2}[(n_1n_2^2 - 4m_2n_2)M_1(G_1) + M_1(G_2)M_1(G_1) + (n_2n_1^2 - 4m_1n_1)M_1(G_2) + 8m_1m_2n_1n_2]^2$$

$$
\begin{aligned}
-\frac{1}{2}[n_1Y(G_2)[n_1^4 + 6n_1M_1(G_1) - 8n_1^2m_1 - 4F(G_1)] + 4n_1^2n_2F(G_2)[2n_1m_1 - 3M_1(G_1)]& \\
+ n_2Y(G_1)[n_2^4 + 6n_2M_1(G_2) - 8n_2^2m_2 - 4F(G_2)] + 4n_2^2n_1F(G_1)[2n_2m_2 - 3M_1(G_2)] + Y(G_1)Y(G_2)& \\
+ 12n_1n_2F(G_1)F(G_2) + 6n_1^2n_2^2M_1(G_1)M_1(G_2)]& \\
- [HM_2(G_2)[n_1(n_1^5 + 16n_1m_1^2 - 10n_1^3m_1 - 8n_1M_2(G_1) - 2n_1F(G_1) + 4ReZG_3(G_1))& \\
+ M_1(G_1)(M_1(G_1) + 6n_1^3 - 8n_1m_1)] + n_2^2F(G_2)M_1(G_1)[n_1^3 - 4n_1m_1 + M_1(G_1)]& \\
+ HM_2(G_1)[n_2(n_2^5 + 16n_2m_2^2 - 10n_2^3m_2 - 8n_2M_2(G_2) - 2n_2F(G_2) + 4ReZG_3(G_2))& \\
+ M_1(G_2)(M_1(G_2) + 6n_2^3 - 8n_2m_2)] + n_1^2F(G_1)M_1(G_2)[n_2^3 - 4n_2m_2 + M_1(G_2)]& \\
+ 2n_2ReZG_3(G_2)[n_1^2(2n_1^2m_1 + F(G_1) - 8m_1^2 + 4M_2(G_1)) - M_1(G_1)(M_1(G_1) + 2n_1^3 - 6n_1m_1)]& \\
+ 4n_2^2M_2(G_2)[4n_1^2m_1^2 + M_1^2(G_1) - 4n_1m_1M_1(G_1)]& \\
+ 2n_1ReZG_3(G_1)[n_2^2(2n_2^2m_2 + F(G_2) - 8m_2^2 + 4M_2(G_2)) - M_1(G_2)(M_1(G_2) + 2n_2^3 - 6n_2m_2)]& \\
+ 4n_1^2M_2(G_1)[4n_2^2m_2^2 + M_1^2(G_2) - 4n_2m_2M_1(G_2)]& \\
+ 2M_1(G_1)M_1(G_2)[n_1^3(2n_2m_2 - M_1(G_2)) + n_2^3(2n_1m_1 - M_1(G_1))] + n_1^4m_1M_1^2(G_2) + n_2^4m_2M_1^2(G_1)& \\
- 6n_1n_2ReZG_3(G_1)ReZG_3(G_2) - n_1^2n_2^2[F(G_1)F(G_2) + 8M_2(G_1)M_2(G_2)] - 2HM_2(G_1)HM_2(G_2)].&
\end{aligned}
$$

Proof.

The proof follows from the relations $\overline{HM}_2(G) = \frac{1}{2} M_1^2(G) - \frac{1}{2}Y(G) - HM_2(G)$, given in Proposition 3.1, and by replacing each G with $G_1 \vee G_2$, which yield

$$\overline{HM}_2(G_1 \vee G_2) = \frac{1}{2} M_1^2(G_1 \vee G_2) - \frac{1}{2}Y(G_1 \vee G_2) - HM_2(G_1 \vee G_2),$$

and $M_1(G_1 \vee G_2)$, given in Theorem 3.4 in [31]. $Y(G_1 \vee G_2)$, given in Theorem 2.5 in [13]. and $HM_2(G_1 \vee G_2)$, given in (Theorem 3.5 in [16]).

Theorem 3.3 The Hyper-Zagreb coindex of $G_1 \oplus G_2$ is given as:

$$
\begin{aligned}
\overline{HM}_2(G_1 \oplus G_2)& \\
= \frac{1}{2}[(n_1n_2^2 - 8m_2n_2)M_1(G_1) + 4M_1(G_1)M_1(G_2) + (n_2n_1^2 - 8m_1n_1)M_1(G_2) + 8m_1m_2n_1n_2]^2& \\
- \frac{1}{2}[n_1Y(G_2)[n_1^4 + 24n_1M_1(G_1) - 16n_1^2m_1 - 32F(G_1)] + 8n_1^2n_2F(G_2)[n_1m_1 - 3M_1(G_1)]& \\
+ n_2Y(G_1)[n_2^4 + 24n_2M_1(G_2) - 16n_2^2m_2 - 32F(G_2)] + 8n_2^2n_1F(G_1)[n_2m_2 - 3M_1(G_2)] + 16Y(G_1)Y(G_2)& \\
+ 48n_1n_2F(G_1)F(G_2) + 6n_1^2n_2^2M_1(G_1)M_1(G_2); HM_2(G_1[G_2])& \\
= n_2^6HM_2(G_1) + n_1HM_2(G_2) + n_2^4m_2[Y(G_1) + 4ReZG_3(G_1)] + 4n_2m_1ReZG_3(G_2) + 3n_2^2F(G_1)M_1(G_2)& \\
+ n_2^2M_1(G_1)[F(G_2) + 4M_2(G_2)] + m_1M_1^2(G_2) + 4n_2m_2[4n_2m_2M_2(G_1) + M_1(G_1)M_1(G_2)]]& \\
- [HM_2(G_2)[n_1(n_1^5 + 64n_1m_1^2 - 20n_1^3m_1 - 64n_1M_2(G_1) - 16n_1F(G_1) + 64ReZG_3(G_1))& \\
+ M_1(G_1)(16M_1(G_1) + 24n_1^3 - 64n_1m_1)] + n_2^2F(G_2)M_1(G_1)[n_1^3 - 8n_1m_1 + 4M_1(G_1)]& \\
+ HM_2(G_1)[n_2(n_2^5 + 64n_2m_2^2 - 20n_2^3m_2 - 64n_2M_2(G_2) - 16n_2F(G_2) + 64ReZG_3(G_2))& \\
+ M_1(G_2)(16M_1(G_2) + 24n_2^3 - 64n_2m_2)] + n_1^2F(G_1)M_1(G_2)[n_2^3 - 8n_2m_2 + 4M_1(G_2)]& \\
+ 4n_2ReZG_3(G_2)[n_1^2(2n_1^2m_1 + 2F(G_1) - 8m_1^2 + 8M_2(G_1)) - M_1(G_1)(4M_1(G_1) + 2n_1^3 - 12n_1m_1)]& \\
+ 16n_2^2M_2(G_2)[n_1^2m_1^2 + M_1^2(G_1) - 2n_1m_1M_1(G_1)]& \\
+ 4n_1ReZG_3(G_1)[n_2^2(n_2^2m_2 + 2F(G_2) - 8m_2^2 + 8M_2(G_2)) - M_1(G_2)(4M_1(G_2) + 2n_2^3 - 12n_2m_2)]& \\
+ 16n_1^2M_2(G_1)[n_2^2m_2^2 + M_1^2(G_2) - 2n_2m_2M_1(G_2)]& \\
+ 4M_1(G_1)M_1(G_2)[n_1^3(n_2m_2 - M_1(G_2)) + n_2^3(n_1m_1 - M_1(G_1))] + n_1^4m_1M_1^2(G_2) + n_2^4m_2M_1^2(G_1)& \\
- 48n_1n_2ReZG_3(G_1)ReZG_3(G_2) - 2n_1^2n_2^2[F(G_1)F(G_2) + 8M_2(G_1)M_2(G_2)] - 64HM_2(G_1)HM_2(G_2)].&
\end{aligned}
$$

Proof.

Similarly, as theorem 3.2, we obtain the required.

Conclusion

In this paper, we have investigated some of the basic mathematical properties of the second Hyper-Zagreb coindex and obtained explicit formula for their values under graph operations, exactly disjunction and symmetric difference.

Authors Contribution

All authors discussed the results and contributed to the final manuscript.

Data Availability

All the data supporting the results are included in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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